William Schell
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Cog 356
Chapter 7 Problem Set

1. Draw a set of recursive transition networks which define well-formed formulas in the propositional calculus.

- Recursive Formation Rules:

a) ~x

b) < x ∧ y >

c) < x V y >

d) < x ⊃ y >

2. Thinking of the propositional calculus in the terms that Hofstadter presents it, that is, as the formal system he constructs in the chapter:

a. How many axioms in the formal system?

- An infinite number

b. How many rules in the formal system?

- 9

c. What are the names that he gives to these rules?

- Joining Rule
- Separation Rule
- Double-Tilde Rule
- Fantasy Rule
- Carry-over Rule
- Rule of Detachment
- Contrapositive Rule
- De Morgan's Rule
- Switcheroo Rule

d. What is the one rule that you absolutely must use if you are to derive a theorem in this system?

- Fantasy Rule

3. Write down each of the rules of the system, just as Hofstadter does on page 187.

- JOINING RULE: If x and y are theorems, then < x ∧ y > is a theorem.
- SEPARATION RULE: If < x ∧ y > is a theorem, then both x and y are theorems.
- DOUBLE-TILDE RULE: The string '~~' can be delete

from any theorem and can also be inserted into any theorem, provided that the result string is itself well-formed.
- FANTASY RULE: If y can be derived when x is assumed to be a theorem, then < x ⊃ y > is a theorem.
- CARRY-OVER RULE: Inside a fantasy, any theorem from the "reality" c level higher can be brought in and used.
- RULE OF DETACHMENT: If x and < x ⊃ y > are both theorems, then y is a theorem.
- CONTRAPOSITIVE RULE: and are interchangeable
- DE MORGAN'S RULE: and ~< x ∨ y > are interchangeable.
- SWITCHEROO RULE: and are interchangeable.

4. Derive: < < < P ∧ Q > ∧ R > ⊃ < P ∧ < Q ∧ R > > >

[	push
<< P ∧ Q > ∧ R >	premise
< P ∧ Q >	separation
R	separation
P	separation
Q	separation
< Q ∧ R >	joining
< P ∧ < Q ∧ R > >	joining
]	pop
<<< P ∧ Q > ∧ R > ⊃ < P ∧ < Q ∧ R >>>	fantasy

5. Derive: < < P ∨ Q > ⊃ < Q ∨ P > >

[	push
< Q ∨ P >	premise
Q	separation
P	separation
< P ∨ Q >	joining
]	pull
< < P ∨ Q > ⊃ < Q ∨ P > >	fantasy

6. Derive a theorem in the propositional calculus that you think is a little bit interesting, one that neither I asked you to derive nor Hofstadter derived in his book.

7. As Hofstadter mentions mid-way through the chapter, there is a decision procedure for WFFs in the propositional calculus, the method of truth tables. L earn what this method entails, if you are not already clear on that, and write a description of the method that is clear and complete enough that one could easily apply it by referencing your description. That is, describe the process featuring truth tables by which one could determine whether or not a WFF is a theorem in the propositional calculus.

- A truth table shows a set of combinations of values of variables and logic operators from a logical statement or argument. For example, if we are given < P V Q > we would right the permutations as follows:

V	P	Q
True	True	True
True	True	False
True	False	True
False	False	False

Green is the answer to the truth values of the statement < P V Q > (P or Q).

- In a WFF theorem in propositional calculus, it is a theorem if it is a valid argument, meaning if it has two or more premises and they are both true, therefore the conclusion will never be false.

8. Using the truth table based decision procedure, show that the heads will be cut off! Perhaps I should say a bit more. I'm referring to the section on Ganto's Ax. And I'm asking you to show by means of a truth table that the following WFF is a theorem:

< < < P ⊃ Q > ∧ < ~ P ⊃ Q > > ⊃ Q >

P	Q		<	<	<	P	⊃	Q	>	∧	<	~	P	⊃	Q	>	>	⊃	Q	>
T	T					T	T	T		T		F	T	T	T			T	T	
T	F					T	F	F		F		F	T	T	F			T	F	
F	T					F	T	T		T		T	F	T	T			T	T	
F	F					F	T	F		F		T	F	F	F			T	F	

Step 1: Blue
Step 2: Gray
Step 3: Blue ∧ Gray
Step 4: Yellow ⊃ Red
Outcome: Green

9. Choose another interpretation for P and Q in Ganto's statement—one that doesn't involve heads or axes. Write down the words for your proposition P. Write down the words for your proposition Q. Write down a sentence corresponding to Ganto's statement (what he says to the praying monks) under your interpretation.

P: It is cloudy
Q: There will be rain
Sentence: If it is cloudy, there will be rain; If it not cloudy, there will still be rain.

10. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Shortcuts and Derived Rules.

- While working within a system there are patterns that can be found that allow us to make shortcuts or use the derived rules. Even though these rules are not defined in the system it does not mean that we cannot implement them.

11. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Formalizing Higher Levels.

- The idea of creating metatheories based of the implementation of derived rules and using shortcuts, it would allow for people to create

another metatheory from the other metatheory creating a metametatheory which would get a person too far away from the system itself.

12. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Reflections on the Strengths and Weaknesses of the System.

- Propositional calculus is a very simple and precise idea of solving logic. Even though one small mistake can throw off the entire outcome of the problem, by

13. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Proofs vs Derivations.

- Proofs and derivations are very useful in solving the outcomes of propositional logic. However, since these are complex it is easy for a user to make a mistake while solving them out which could lead to an incorrect outcome.

14. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled The Handling of Contradictions.

- When a contradiction arises, it is important, and very difficult, to figure out where a person went wrong. Once a contradiction occurs there must be a reasonable way to fix it, either by reproducing the proof or derivation, or going back looking through the work and figuring out if there is an error within the system itself.

15. In one paragraph, write your reaction to this chapter.

- This chapter was extremely triggering to me. This chapter really brought me back to when i was a student at Onondaga Community College and taking a philosophy class about logic. Proofs and derivations of logic are not always the easiest things to wrap your head around. There are so many different factor and rules and ideas that go into propositional logic that it is very fun but very difficult at the same time. I found that the way the material and ideas were presented were very clear and easy to understand. Propositional calculus compared to formal systems, that we have read about before, are much more difficult. There are still rules, but instead of a true system where there is a start and, most likely, a definite end, in propositional logic there are many different ways to approach the same problem.