pq-system

Questions related to Chapter 2 from GEB. This has to do with meaning and form in mathematics. We see this used in the pq-system.

Will Schell
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Q1. What is the formal system of Chapter 2 called?

- The formal system is the pq-system.

Q2. What are the distinct symbols of this formal system?

- The distinct symbols are p, q, and –

Q3. How many axioms in the pq-system?

- There are an infinite number of axioms.

Q4. Write down the axiom schema for the pq-system.

- The axiom schema is xp - qx

Q5. What is a "schema"? Define the term.

- A schema is a mold in which something is cast.

Q6. Write down the three shortest axioms in the pq-system.

- The three shortest axioms in the pq-system are:

1. - p - q - -

2. - - p - q - - -

3. - - - p - q - - - -

Q7. Write down the sole rule of production for the pq-system.

- "Suppose x, y, and z all stand for particular strings containing only hyphens. And suppose that xpyqz is known to be a theorem. Then xpy - qz is a theorem."

Q8. Show that --p---q----- is a theorem of the pq-system. That is, derive it from an axiom and repeated application of the rule.

(1) -p-q-- axiom.

(2) --p-q--- from (1)

If --p--q---- turns out to be a theorem then --p---q----- can be derived because a hyphen was added in front of the q and behind the q, which is the rule to produce new theorems.

Q9. Show that -----p----q--------- is a theorem of the pq-system. That is, derive it from an axiom and repeated application of the rule.

(1) -----p-q------ axiom

(2) -----p--q------- from (1)

(3) -----p---q-------- from (2)

(4) -----p----q--------- from (3)

Q10. Write down a string of symbols in the pq-system which is not well formed.

--p--q-p---p-----q

Q11. State a decision procedure for the pq-system.

-   The first two hyphen groups should add up in length to the third hyphen-group

Q12. In the longest paragraph on page 48, Hofstadter engages in some "top-down" reasoning. In one sentence, articulate exactly what it is that he demonstrates with his top-down reasoning in the paragraph.

-   Hofstadter demonstrates that, with the rules of a system being known, one can apply the rules in reverse order to determine the original axiom or string (non-axiom) that yielded a given string and thus discover the processes and mechanics of the system.

Q13. In one sentence, characterize "top-down" reasoning.

-   Start with the theorems and find the axioms.

Q14. In one sentence, characterize "bottom-up" reasoning.

-   Start with the axioms and get to the theorems.

Q15. Consider the procedure for generating theorems of the pq-system given at the top of page 49. What will be in the bucket after executing statements (1a) and (1b) and (2a) and (2b) and (3a) and (3b) -- after all six of these statements have been executed!

Each of these will be produced after all six of Hofstadter's statements are executed:

-p-q---p--q-----p-q-----p--q-------p-q-------p--q-----

Q16. What role does the procedure introduced on the top of page 49 play in Hofstadter's presentation of the pq-system and related matters? Answer in just one sentence!

-   It shows the possibilities of the structures involving shortening and lengthening of strings.

Q17. What is an isomorphism?

-   An information-preserving transformation that applies when two intricate structures are able to be mapped onto each other, where there is a counterpart in the other structure.

Q18. What is an interpretation in the context of a formal system?

-   In the context of a formal system, an interpretation is a symbol-word correspondence.

Q19. When was Linear B deciphered?

-   Linear B was accurately and completely deciphered in 1953 by Michael Ventris

Q20. How many meaningful interpretations of the pq-system did Hofstadter present in this chapter?

- 2

Q21. How many meaningless presentations of the pq-system are there?

- Infinite amount of meaningless presentations

Q22. In 50 plus or minus 20 words, summarize what Hofstadter says in the section titled "Formal Systems and Reality".

- Reality is one, big, formal system. This meaning that everything around us is composed of symbols all around us. Symbols and rules are all around us in reality and are building blocks for how we as humans survive and live our lives. With the exception of some ideas of physics there is some doubt of a formal system being linked to how reality functions.