Tq-C-P-A-B-@|-Problem Set

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Problem set from Chapter 3 GEB

1) Write down the axioms schema and the three shortest axioms in the tq-system

- The axiom is:

xt-qx, whenever x is a hyphen-string.

- The three shortest axioms are:

a) t-q

b) -t-q-

c) --t-q--

2) Write down the sole rule of inference for the tq-system and apply it to the well-formed string -----t-----q-----

- Rule of Inference: Suppose that x, y, and z are all hyphen-strings and suppose x ty qz is an old theorem. Then xty-qzx is a new theorem.

-----t-----q----- becomes -----t------q----------

3) Reasoning in I-mode, argue that the string you produced in the previous item is not a theorem in the tq-system.

- This is not a theorem because the hyphen string x multiplied by hyphen strings y does not equal the hyphen string z.

4) Working in the M-mode, show that -----t---q--------------- is a theorem in the tq-system.

1) -----t-q----- (axiom)

2) -----t--q---------- (from 1 by the rule of inference)

3) -----t---q--------------- (from 2 by the rule of inference)

5)What are the two rules of the C-system?

a) Suppose x, y, and z are hyphen strings. If x-ty-qz is a theorem, then C z is a theorem.

b) Suppose x is a hyphen-string. If Cx is not a theorem, then Px is a theorem.

6) Working within the C-system, argue that C-------- is a theorem of the system.

1) ----t-q---- (axiom)

2) ----t--q-------- (from 1 by rule)

3) C-------- (from 2 by rule 1 in C- system)

7) Does adding the following rule to the C-system constitute a Post production system for determining primes? Please explain your response.

- Suppose x is a hyphen string. If Cx is not a theorem, then Px is a theorem. The addition of the proposed rule does not create a Post production system due to it not allowing for manipulation of strings.

8) Hofstadter writes: When a figure or "positive space" (e.g., a human form, or a letter, or a still life is drawn inside a frame, an unavoidable consequence is that its complementary shape-also called the "ground", or "background", or "negative space"-has also been drawn.

According to this view, the quiche pan shown below that I computationally rendered would be considered negative space. Explain how this is so.

- The white space is considered as negative space because black shapes, also known as the positive space, are between the border and the middle.

9) Consider the A-system as defined by the following axiom and theorem:

- Axiom: A--

- Rule: Suppose that x is a hyphen-string. If Ax is a theorem, so is Ax--

Answer the following questions with respect to the A-system:

a) Show that A-------- is a theorem of the A-system by working within the system

1) A-- (axiom)

2) A---- (from 1 by the rule)

3) A------ (from 2 by the rule)

4) A-------- (from 3 by the rule)

b) Specify a decision procedure for determining theoremhood in the A-system.

- If the number of hyphens must be even.

c) Provide an I-mode argument that the string A---------- is not a theorem of the A-system.

- If there is an odd number of hyphens, then it is not a theorem.

d) What subset of the natural numbers do you think it was my intent to capture with the A-system?

- All positive and even integers above 2 are captured within the A-system.

10) Consider the as yet to be formally defined by B-system which you should imagine is intended to capture precisely all of the natural numbers that the A-system does not capture.

a) Propose, by analogy with the rule on page 66 of GEB, an invalid rule for producing theorems in the B-system.

- Suppose x is a hyphen string. If Ax is not a theorem, then Bx is a theorem

b) Define a (valid) Post production system for the B-system in terms of the one axiom and one rule

- Axiom: B-

- Rule: Suppose that x is a hyphen-string. If Bx is a theorem, so is Bx--

c) Derive B---------- within the B-system

1) B- (axiom)

2) B--- (from 1 by the rule)

3) B----- (from 2 by the rule)

4) B------- (from 3 by the rule)

5) B--------- (from 4 by the rule)

6) B---------- (from 5 by the rule)

d) What subset of natural numbers does the B-system capture?

- All odd numbers.

11) Under interpretation, what does the A-system theorem A-------- say? Under interpretation what does the B-system theorem B---------- say?

- The given A-system theorem says that 8 is an even number.

- The B-system theorem says that 11 is an odd number.

12) What does it mean for a set to be "recursively enumerable?" What does it mean for a set to be "recursive?"

- "recursively enumerable" means that a set of strings can be generated by typographical rules

- A set is "recursive" if the background of a set is a figure on its own.

13) Argue that the set of even numbers is recursively enumerable.

- The set of even numbers is recursively enumerable because it can be generated by typographical rules.

14) Argue that the set of even numbers is recursive.

- The set of even numbers, A-system, is recursive because the set of odd numbers, B-system, is also recursively enumerable.

15) Argue that the set of prime numbers is recursively enumerable.

- The set of prime numbers is recursively enumerable because it can be generated by typographical rules.

16) Argue that the set of prime numbers is recursive.

- The set of prime numbers is recursive because the set of non-prime numbers is also recursively enumerable.

17) In a sentence or two, explain why you think that I am not asking you in this problem set to derive something like P-----with the P-system.

- To derive a theorem of the P-system one would first have to check each number for whether it is divisible or not.

18) Consider the following post production system, a system that we call the @|-system. Answer the following questions about the @|-system:

a) How many axioms in the system? How many rules in the system? How many theorems in the system?

- One axiom

- 34 rules

- Infinite theorems

b) Show that the @||-|||-||-|||- is a theorem of the @|-system performing derivation within the system.

1) @||-|||- (axiom)

2) @||-|||-| (from 1 by rule 6)

3) @||-|||-|| (from 2 by rule 5)

4) @||-|||-||- (from 3 by rule 2)

5) @||-|||-||-| (from 4 by rule 7)

6) @||-|||-||-|| (from 5 by rule 5)

7) @||-|||-||-||| (from 6 by rule 3)

8) @||-|||-||-|||- (from 7 by rule 1)

c) Show that CDEFGABC is a theorem of @|-system by performing a derivation within the system.

1) @||-|||- (axiom)

2) C||-|||- (from 1 by rule 8)

3) CD|-|||- (from 2 by rule 11)

4) CDE-|||- (from 3 by rule 15)

5) CDEF|||- (from 4 by rule 20)

6) CDEFG||- (from 5 by rule 21)

7) CDEFGA|- (from 6 by rule 25)

8) CDEFGAB- (from 7 by rule 29)

9) CDEFGABC (from 8 by rule 34)

d) Show that GABCDEXG is a theorem of @|-system by performing a derivation within the system.

1) @||-|||- (axiom)

2) G||-|||- (from 1 by rule 9)

3) GA|-|||- (from 2 by rule 25)

4) GAB-|||- (from 3 by rule 29)

5) GABC|||- (from 4 by rule 34)

6) GABCD||- (from 5 by rule 11)

7) GABCDE|- (from 6 by rule 15)

8) GABCDEX- (from 7 by rule 19)

9) GABCDEXG (from 8 by rule 24)

e) Show that DEXGABVD is a theorem of @|-system by performing a derivation within the system.

1) @||-|||- (axiom)

2) D||-|||- (from 1 by rule 10)

3) DE|-|||- (from 2 by rule 15)

4) DEX-|||- (from 3 by rule 19)

5) DEXG|||- (from 4 by rule 24)

6) DEXGA||- (from 5 by rule 25)

7) DEXGAB|- (from 6 by rule 29)

8) DEXGABV- (from 7 by rule 33)

9) DEXGABVD (from 8 by rule 14)

f) What do you think I had in mind when I invented the system?

- It looks like it would be a form of musical representation