

Will Schell

Propositional Logic Problem Set 1

1. Jacques Herbrand paragraph

- Jacques Herbrand was born in 1908 and died in 1931, at age 23. He was already considered one of the greatest mathematicians by his professors. He helped come up with theories in mathematical logic. He is known for his theory known as the Herbrand's theory. This theory is made up of two parts, 1 part being from if proof theory doctoral thesis and the other half from the Herbrand-Ribet theorem.

2. Alfred Horn paragraph

- Alfred Horn was born in 1918 and died in 2001. He was a mathematician notable for his work in lattice theory and universal algebra. In a paper that Horn wrote he describes two things called, Horn Clauses and Horn sentences. The Horn clause and Horn sentence would later form the foundation of logical programming.

3. John Alan Robinson paragraph

- John Alan Robinson was born in 1930 and died in 2016. Robinson helped with automated theorem proving. He made a unification algorithm which prepared the ground for logic programming paradigm, in particular the Prolog language.

4. Suppose

- * $P ::$ she needs a doctor
- * $Q ::$ she needs a lawyer
- * $R ::$ she had an accident
- * $S ::$ she is sick
- * $U ::$ she is injured

State the following formulas in English

- $(S \rightarrow P) \wedge (R \rightarrow Q)$
 - If she is sick, then she needs a doctor, and, if she has an accident, then she needs a lawyer.
- $P \rightarrow (S \vee U)$
 - She is sick or injured if she needs a doctor.
- $(P \wedge Q) \rightarrow R$
 - If she needs a doctor or a lawyer, then she had an accident.
- $(P \wedge Q) \leftrightarrow (S \wedge U)$
 - She needs a doctor and a lawyer if and only if she is sick and she is injured.
- $\sim (S \vee U) \rightarrow \sim P$

- If she is not sick and she is not injured then she does not need a doctor

5. define an interpretation of a wff.

- interpretation of a wff is an assignment of truth value to an atom, such as G.

6. How many interpretations of 4 atoms?

- $2^4 = 16$

7. What does it mean if an interpretation satisfies a wff?

- If formula is true under interpretation F we say F satisfies the formula.

8. Write down an interpretation and a wff with three different atoms for which the interpretation satisfies the wff.

- WFF = $(F \vee G) \rightarrow H$
- FGH = TTT
- $(T \vee T) \rightarrow T$
- $T \rightarrow T$
- T

9. What does it mean for an interpretation to falsify a wff?

- falsifying a wff is when an formula is false under the interpretation of an atom.

10. Write down an interpretation and a wff with two different atoms for which the interpretation falsifies the wff

- WFF = $(F \vee G) \rightarrow H$
- FGH = TTF
- $(T \vee T) \rightarrow F$
- $T \rightarrow F$
- F

11. When is an wff valid?

- a wff is valid when all interpretations of a formula are true.

12. wff with 3 atoms that is valid.

F	G	H		$(G \vee H)$	$F \wedge (G \vee H)$	$F \wedge (G \vee H) \rightarrow G$
T	T	T		T	T	T
T	T	F		T	T	T
T	F	T		T	T	T
T	F	F		F	F	T
F	T	T		T	F	T
F	T	F		T	F	T
F	F	T		F	F	T

F	F	F		F		F		T
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13. inconsistent wff?

- a wff that's interpretations are a mix of true and false

14. 2 atoms to show inconsistent

F	G		$F \vee G$
T	T		T
T	F		T
F	T		T
F	F		F

15. Disjunctive norm form.

- f is in disj. norm form if F has form $F_1 \vee \dots \vee F_n$ where each F_i is a conjunctive literal

17. Conjunctive norm form.

- F is in conj. norm form if F has form $F_1 \wedge \dots \wedge F_n$ where each F_i is a disjunctive literal